From Throw to Flow, and Back
Fountains for Research and Teaching in 18th-Century Leiden

JIP VAN BESOUW / PIETER T. L. BECK

Introduction

The Leiden Cabinet of Physics, one of the largest collections of eighteenth-century scientific instruments, contains a surprisingly substantial set of fountains. One might think that these instruments have something to do with Baroque gardens and their exquisite fountains. However, aesthetics was not of great importance to the Leiden fountains: they do not produce particularly elegant or unexpected jets of water. Instead, they are bulky machines with large water reservoirs in plain sight. What they do, is produce jets of water of specific parabolic shapes, of which the direction and velocity can be varied. Clearly, Willem Jacob 's Gravesande (1688–1742) and Petrus van Musschenbroek (1692–1761), the two key professors in the history of the Leiden Cabinet of Physics, had specific uses in mind for these fountains.

In this chapter, we analyse how these instruments were in fact used in the work of the two professors. We do so by studying the instruments and the documents in which they were discussed side by side. Our key finding is that the fountains were of importance both for teaching and for further investigations into fluid flow. Thus, the fountains straddled the divide between 'teaching' and 'research'. For 's Gravesande, the main use of these fountains was in the investigation of forces and resistances in water flow, a topic which he inherited from Isaac Newton and to which he dedicated a significant part of his written work. Van Musschenbroek, on the other hand, used these fountains in his teaching on projectile motion in general. Instead of investigating the specific properties of water in motion, Van Musschenbroek used water to visualize parabolic shapes. He directly linked experiments with fountains to the diagrams in the textbook. As such, these experiments both confirmed and further elucidated the content found in the textbook and contribute to the development of what we will call "diagrammatic literacy". In this way, we analyze how the instruments were used for conceptual clarification, both in research and in educational contexts.

This combination of teaching and research points us to a reconsideration of a historiographical dichotomy. The consensus among historians is that 's Gravesande and van Musschenbroek were mainly popularisers of Newton's work and added little of original value themselves (Jorink & Zuidervaart, 2012, pp. 35–36, 45; Heilbron, 2011, pp. 176–177). Although it is known that some of their instruments had additional purposes, there seems to be a consensus that they were teaching devices first. By manifesting, for instance, the laws of mechanics with well-developed experiments, they set those laws directly before the eyes of their audience. In this way, 's Gravesande and van Musschenbroek are supposed to have convinced their students of the new mechanics without having to give a full mathematical demonstration of these laws (Hooijmaijers & Maas, 2013). While this idea of instruments for simplification rests partly on their own discourse (see their letters to Newton printed in Hall, 1982, pp. 26, 32), it is misleading.

Nevertheless, teaching is certainly part of the history of the Leiden Cabinet of Physics. The instruments designed and used by 's Gravesande and van Musschenbroek were in general presented in works such as the *Physices elementa mathematica* and *Institutiones physicae*, books which intended to give an accessible overview of natural philosophy in general. These books have generally been presumed to be textbook on "Newtonian physics" and many of the instruments depicted in them became prototypes in physics education after 's Gravesande and van Musschenbroek introduced them to their readers.

In this chapter, we want to go beyond this narrative. We align ourselves with recent literature that has pointed towards complex and often fruitful interactions between teaching and research. Recently, there has been a renewed interest in science pedagogy in the historiography of science (Mody & Kaiser, 2008; Kaiser, 2005). This literature provides numerous examples in which teaching fed into research activities. Perhaps the most well-known example is the way Mendelejev's formulation of the periodic law was an immediate result of his work on a new chemistry textbook (Robinson, 2019; Gordin, 2019). Similarly, Andrew Warwick has analysed the entanglement between developments in the teaching practices in mathematics at Cambridge University and developments in mathematical theoretical physics (2003).

The eighteenth century was no exception to this. In his work on Boerhaave, a central figure at Leiden in the early eighteenth century, John C. Powers has pointed out that 'Boerhaave's *Elementa Chemiae* functioned equally as a pedagogical text for students and as an account of more advanced work addressing fundamental questions [...] This dual function would have surprised no one in the eighteenth century, when university professors [...] often communicated their discoveries and innovations in academic genres: textbooks and dissertations' (2012, pp. 3–4). Kathryn M. Olesko has made the more general argument that although 'we have come to think of the nineteenth century as holding the clues to all aspects of the development of modern science, including its pedagogical dimensions […] the eighteenth century may actually have more to offer us in terms of *why* (rather than *how*) these changes took place' (2006, p. 876). According to her, the production of textbooks played a crucial role therein.

In line with these recent developments in the literature, we will argue that the demonstration instruments used by 's Gravesande and van Musschenbroek were "objects of understanding" in a broad sense. They were specifically built for the purposes of conceptual clarification, but their function was not necessarily limited to a pedagogical one.

Pissing machine prehistory[1]

In this section, we give a brief overview of the historical context, from Torricelli to Newton, of our fountains and the conceptual developments they were involved in. This historical overview will show that these fountains, as well as the jets of water they produced, were multidimensional objects from the very beginning, connected not just to the mathematical modelling or theorizing of hydromechanics, but also to various practical concerns such as the motion of water in rivers and in actual fountains. Moreover, these objects were often explicitly linked to the science of mechanics in a much broader sense.

The question that led to the line of instruments with which we are concerned was how fast water would flow out of a pierced tank of water. The efflux problem, as we will call it, attempted to quantify the velocity of water spouting out of the orifice of that tank. The efflux problem became topical in the 1620s. Benedetto Castelli (1578–1643) studied it after he was asked to the Papal States to give advice on plans to divert rivers in the North of Italy. The motion of water in an actual river is, of course, a very complex affair. As Castelli was first and foremost a mathematician, not generally concerned with practical water management, he set out to understand rivers by modelling simple flows. With his friend and collaborator Galileo, Castelli set out to quantify flow out of pierced cisterns, in which there is just one jet of water to study (Maffioli, 1994, pp. 41–78; Omodeo, 2022, pp. 550–554; Bertoloni Meli, 2006, pp. 80–85).

Yet Castelli and Galileo did not produce an acceptable general solution to the efflux problem. The real breakthrough came with Evangelista Torricelli (1608–1647), a former student of Castelli and collaborator of Galileo. Torricelli's account of the problem of outflow seems to have been a direct consequence of his dissatisfaction with earlier treatments (Maffioli, 1994, pp. 75–76). We do not know what Torricelli's apparatus

1 We take the term 'pissing machine' from Denis Weaire (2022), who asserted that the term is a common descriptor for historical instruments which produce horizontally aimed jets of water.

looked like, since only a diagrammatic image exists. Presumably, a simple tank of water was pierced, a nozzle put into the hole, and the water would spout out. For Torricelli, we know he took a large vessel and made a small hole in it, so that the level of water would drop only slowly.

Whatever the instrument, Torricelli's experiments were successful and the principle now named after him is well known: Torricelli concluded that the velocity of water flowing out at the orifice of a pierced vessel was proportional to the square root of the height of the water level above the orifice. This "proposition", as Torricelli called it, was not reached through induction from empirical facts. Instead, Torricelli came to it by considering water in motion as analogous to bodies in free fall, as studied by Galileo. Torricelli did not measure but *supposed* ["supponimus"] that the water spouting out of the orifice had the same velocity as a drop of water would have acquired if it had freely fallen from the water level in the reservoir to the height of the orifice. This shows that Torricelli's studies were deeply connected with the problem of free fall: following Galileo, Torricelli had focused first on ballistics. Building on Galileo's theoretical description of free fall, the two studied the motions of objects launched into the air and established that such objects, canon balls for example, would need to follow a parabolic shape. A jet of water spouting out of the nozzle in the tank followed a similar path when the nozzle pointed at an angle (Torricelli, 1644, pp. 191–198; Bertoloni Meli, 2006, pp. 128–129; Blay, 2007, pp. 13–14; Maffioli, 1994, pp. 81–84.)

Proof of the viability of the analogy, for Torricelli, was that water spouting out vertically would almost reach the height of the water level in the tank. If it got to the same level exactly this would mean, under free fall, that the starting velocity was that which Torricelli had 'supposed'. Torricelli attributed the discrepancy between the calculated height and the actual height of a jet of water to resistance of the air and the urge of the water on top of the jet to fall back on the water beneath it. He asserted that these factors could be made smaller if quicksilver were used, instead of water, to perform the experiment. These consideration on the discrepancies, and the investigations with the water jet instrument more generally, thus helped to figure out that water in motion was within the scope of Galileo's concept of free fall (Torricelli, 1644, p. 92).

That water spouting out of an orifice would not behave regularly was something all authors here considered remarked upon. Torricelli's reference to mercury was an attempt to make the fountains behave a little closer to the theoretical model, which was never actually reached in practice. As a results of this uncertainty, the first two decades after Torricelli saw multiple attempts to decide whether the speed of water flowing through a hole was indeed proportional to the square root of the height of the water standing above that hole, or instead to its height simple (Bertoloni Meli, 2006, pp. 167–178).

Another approach to the irregular behavior of water in motion was taken by the members of the *Académie des Sciences* in Paris, who also investigated the sources of deviations from the predicted values. They did so in response to two enormous water projects undertaken in the 1660s: the digging of the Canal du Midi, connecting the Atlantic and the Mediterranean, and the construction of the fountains of the Palace of Versailles. Both these projects required a solid understanding of the behaviour of running water. In Versailles, located on a hill far above running rivers, getting water running fast enough to feed the fountains was a major challenge (Boschiero, 2020; Mukerji, 2009). Edme Mariotte (1620–1684), Jean Picard (1620–1682), Christiaan Huygens (1629–1695), and other members of the Académie set out to investigate many factors that might come into play in actual water flows, rather than theoretical ones. Their experiments were not only meant to decide whether Torricelli's model held, at least by approximation. Besides that, they also tried to get a grip on the forces and resistances in play. All of their experiments, as well as the conclusion drawn from them, were discussed collectively (Blay, 2007, pp. 52–66; Boschiero, 2020, pp. 1115–1117; Bertoloni Meli 2006, pp. 176–179).

In a noteworthy experiment, published in the acts of the *Académie*, Huygens attempted to quantify the impact of a jet of water. He projected a jet onto a balance, thereby pushing one side down. Using counterweights, he subsequently brought the balance back into equilibrium. Thus, the counterweights gave a measure of what Huygens called the "force" of different jets, a force which was of course related to the velocity of the jet and therefore to the height of the water level in the reservoir. This instrumental setup allowed Huygens to quantify "forces" via the weights of virtual cylinders of water with specific dimensions. Huygens found these virtual weights, which were mathematical constructs, to be proportional to the square of the velocity of the water (Blay, 2007, pp. 58–59).

Mariotte's experiments were equally mathematical, but had more of a practical objective: his experiments discussed at length how the dimensions of pipes and nozzles would influence flow conditions. Many of the experiments were reported in Mariotte's posthumous *Traité du mouvement des eaux*. Among other things, Mariotte established that, if the height of the water in the reservoir was kept stable, the velocity of water flowing out would be directly proportional to the surface area of the orifice. He backed this up with various tables on measured outflow. However, he also noted that larger orifices would sometimes give less water than expected, compared to small ones (Boschiero, 2020, pp.1122–1124; Bertoloni Meli, 2006, pp.179–180).[2] Mariotte pointed to viscosity, air resistance, in the "impulsion" of the water being poured into the reservoir, as causes of the discrepancies. Values for these causes were expressed in proportion to the opening of the orifice and based on reason, not measured exactly (Mariotte, 1686, pp. 272–282). Thus, the gap between the mathematical model and the experimental values was not yet closed.

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2 Bertoloni Meli asserts that tabulated values presented by Mariotte are not actual experimental data; they indeed seem to contain reasoned averages of the data.

Published just one year later, Newton's *Principia* (1687) contains many experiments on the behaviour of fluids. These are relevant for our story even though they do not focus on the efflux problem. After having established the motions and forces that planets experience under Kepler's laws in Book I, Newton showed in Book II that these motions were incompatible with the Descartes' vortex theory, thereby debunking Descartes' cosmology altogether. He did so via a study of the resistance bodies encounter while moving through a fluid. For Newton, the point was to show and prove that, if the cosmos was filled by a sort of fluid mass such as Descartes had supposed, an object moving through it would encounter noticeable resistance. Newton argued that this resistance would set planets on a much more complicated path than Kepler's laws supposed and would therefore not be compatible with the astronomical observational record.[3]

Newton introduced a new conceptual framework for dealing with such forces altogether. The total resistance force, according to Newton, was the sum of resistance due to the fluid's inertia, internal friction of the fluid, and the friction on the surface of the object moving through it. These three terms, he argued, were respectively proportional to the object's velocity squared, to its velocity simple, and independent of velocity. He focused on figuring out which factor would dominate the total resistance under what circumstances. Book II includes various important experiments as well as a theoretical study of projectile motion under various forms of resistance (Smith, 1998). The details of this study need not concern us, but they do show that for Newton, as for others, the study of water in motion was closely related to projectile motion.

In the first edition of the *Principia*, most experiments were carried out with a pendulum moving through water. One experiment, however, used efflux to find a measure for the resistance of a body. Surprisingly, Newton took water spouting out of a hole could only reach halfway to the height of the water level. In the early 1690s, after Huygens and others convinced him he was wrong, Newton carried out more detailed experiments. He dropped balls down a reservoir filled with water, but without an orifice, and obtained lower values for the total resistance. These and other experiments were reported in the second edition of the *Principia* (1713), in which Book II was heavily rewritten (Smith, 2005, pp. 138–140; Newton, 1999 pp. 733–741).

Thus, in the 1710s, the basics of jets of water were well understood but research did not stop. Outflow experiments were being used not only to develop further conceptual work on forces and resistance but also to figure out how real fountains would behave. In other words, the modelling and experimenting with these instruments had both theoretical and practical spin-offs. Nevertheless, the efflux problem was by no means obvious or well known outside the small circle of mathematicians. This is where teach-

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3 On the central argument of Book II and the role of fluid resistance in it, see Smith, 2005, pp. 127–160. Smith lays out various weaknesses within Newton's theory of fluid motion.

ing came into play. To our knowledge, the first use of a fountain that produced horizontal jets of water, a 'pissing machine', in an educational context, is depicted in a pamphlet advertising a course by Francis Hauksbee the Younger (1687–1763) and William Whiston (1667–1752). This pamphlet contains an image of a narrow pipe connected to a larger reservoir, with water spouting out of various holes in the pipe, in parabolic jets. The text rehearses Torricelli's law (as we will call it from here onwards), explaining that things could not be otherwise, "For twice the Quantity running out, with twice the Velocity, implies that the Force or Pressure to be Fourfold, as the Fourfold Altitude requires" (Hauksbee & Whiston, 1714, p. 14). Hauksbee and Whiston thus used Torricelli's law to talk about forces, just as Newton had done the year before. 's Gravesande would do the same, although with much more sophistication.

's Gravesande and the conceptualization of the forces of fluids

's Gravesande was born in 1688, just a year after the Principia first appeared. As the offspring of a landowning family, he followed a conventional path and trained as a lawyer, presumably to take up a function in local governance. From the early 1710s onwards, however, he started building an international reputation as a mathematician, and got in touch with various leaders in the field, most closely with Johan I Bernoulli (1667–1748). He also was one of the founding editors of the *Journal Littéraire*, which discussed scholarly endeavors, mostly in the form of book reviews ranging from theology and literature to mathematics and natural philosophy. In 1715, 's Gravesande was part of an embassy sent to London to discuss the fallout of the War of Spanish Succession. There, he met Newton, was made a Fellow of the Royal Society, and saw various courses on natural philosophy, likely including Whiston and Hauksbee's (Van Besouw, 2016).

In 1717, 's Gravesande was appointed as professor of mathematics and astronomy at Leiden and started teaching various subjects, including natural philosophy. 's Gravesande collaborated with Jan van Musschenbroek (1687–1748), Petrus older brother, to develop a private collection of instruments to use in his teaching. After the death of Wolferd Senguerd (1646–1724), 's Gravesande also became responsible for the university's *Theatrum physicum*. In 1742, after 's Gravesande's death, Leiden University bought his private instruments and the two collections were officially merged (de Clerq, 1987, p. 151).

In 1720, 's Gravesande published the first edition of his magnum opus, the *Physices Elementa Mathematica, Experimentis Confirmata. Sive Introductio ad Philosophiam Newtonianam* (The Mathematical Basics of Physics, Confirmed by Experiments. Or, an Introduction to the Philosophy of Newton). The work went through many editions, with a first abridged version, the *Philosophiae Newtonianae institutiones*, appearing in 1723 and a first enlarged version, more geared towards his peers, in 1725. A final, much enlarged version appeared posthumously in 1742.

Ongoing research on fountains, 1720–1742

The first edition of the *Physices elementa mathematica* is mostly a compendium on mathematical natural philosophy. It contains "books" on mechanics, fluids, optics, and astronomy. For each of these topics, 's Gravesande provided first a general overview of the basics before moving towards discussion of more advanced findings, with the latter mostly coming out of Newton's *Principia* and *Opticks* (1704) (Van Besouw, 2017a).

The instruments with which we are concerned are introduced in chapter seven of the part on hydromechanics, called "On the velocity of fluids, due the pressure of the fluid above it". 's Gravesande's first machine is similar to Hauksbee and Whiston's, although it is an improved version of it [Fig. 1]. The reservoir is much larger, which allows for more stable flow. And instead of on a thin pipe, the nozzles in 's Gravesande's instrument are attached directly to the reservoir. This means turbulence at one of the holes would have little impact on the outflow.

Nothing particularly interesting happens with this machine in the first edition. 's Gravesande first argues for Torricelli's law theoretically and then illustrates it by experiments. His argument is somewhat complicated. It states that the pressure in the water increases proportionally to the depth, and that "quantity of motion" flowing out increases in the same way as the depth of the reservoir. This "quantity of motion" however is "composed" by the velocity and the amount of water flowing out. Therefore, an increase in the pressure would increase both the velocity and the amount of water spouting out, but each only by the square root of the increase in pressure itself. Therefore, it is not the velocity itself, but the velocity squared of the water which is proportional to the height of the water level above the nozzle ('s Gravesande, 1720, I, pp. 127–128).[4]

A second machine [Fig. 2], slender and with a much smaller reservoir, used mercury instead of water. As Torricelli had already suggested, this would lead to a neater jet. 's Gravesande used this instrument to teach the geometry of different jets, with reference to his treatment of ballistics ('s Gravesande, 1720, I, pp. 133–137). To discuss irregularities in fluid flows, 's Gravesande again uses the larger water fountain. In the first edition, he remained general and qualitative (138–142). Following Newton, 's Gravesande also discussed the resistance fluids exerted on bodies moving through them, mentioning two distinct causes: the inertia of the liquid and the cohesion between the internal parts of the liquid. Like Newton, he stated that resistance due to internal cohesion was proportional to the velocity simple, whereas the resistance due to the inertia was proportional to the square of the velocity of a body ('s Gravesande, 1720, I, pp. 118–127).

4 Varignon had made a very similar argument about why Torricelli's law held in 1695 (Blay, 2007, pp. 124–128).

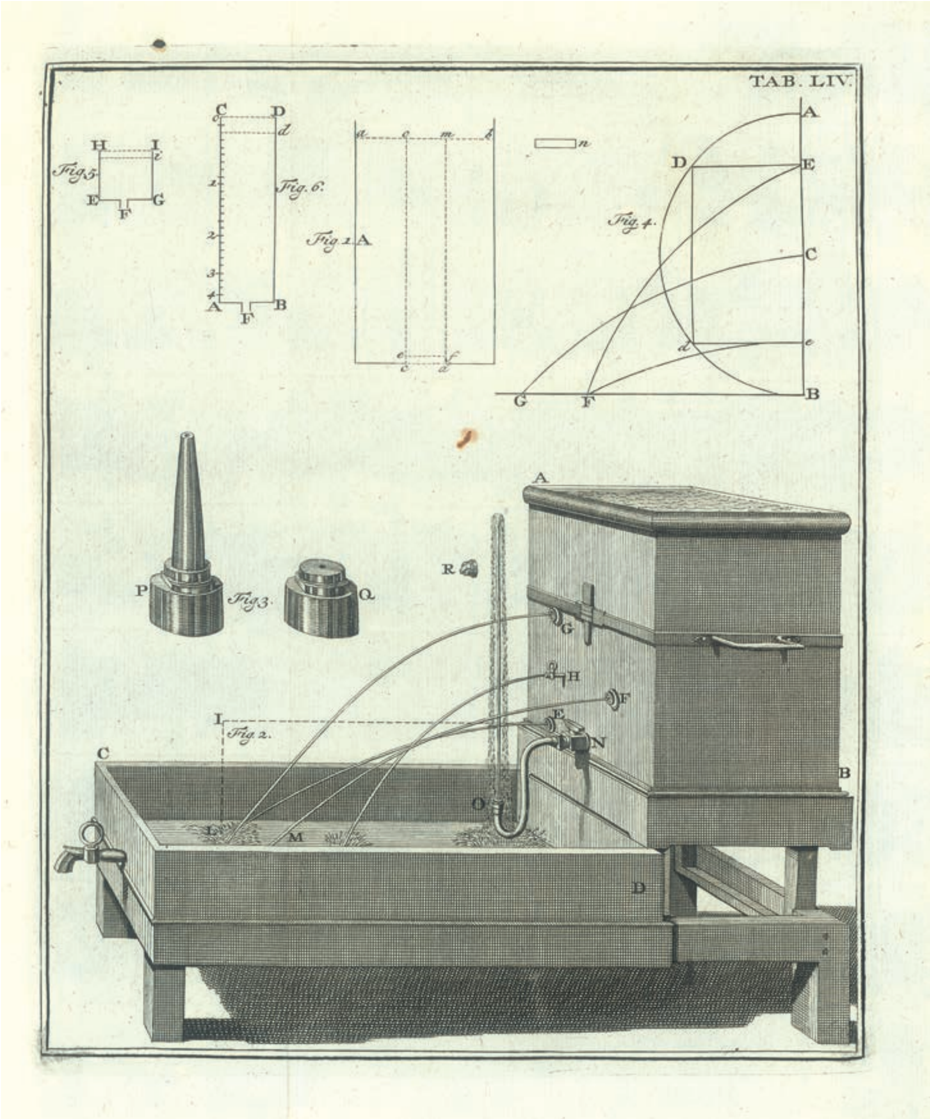

Fig 1 ’s Gravesande’s first fountain, with a large reservoir from which water is projected either vertically or horizontally. ’s Gravesande 1720, I, plate 24.

A lot changes in the parts on fluid motion in the 1725 second edition of ’s Gravesande’s *Physices elementa mathematica*. Whereas the conceptual discussion of the efflux problem in the first edition did not differ much from that of Mariotte, in the second edition emphasis is placed on the concepts of “effects”, “actions” and “forces”. Gone is for example the explanation of Torricelli’s law in terms of the “quantity of motion”. Instead, we now find an explanation in terms of the behavior of individual particles of water

Fig. 2 's Gravesande's mercury fountain. The small reservoir at the top is filled with mercury, which descends through the rod and is projected diagonally into the through.
Photo by Tom Haartsen. Courtesy Rijksmuseum Boerhaave, Leiden.

in the reservoir of the larger machine. 's Gravesande stated that a particle is 'moved by the pressure of all the particles lying around' and 'acquires [its] velocity due to the pressure of the sides.' Things get complicated quickly when we read that 'the action of a fluid on a particle, because of which action velocity is communicated to [the] particle, depends on such a descent of the particles by which the place that had been occupied by the particle is filled' and that 'the force (*vis*) acquired in this descent is completely employed by the action on the moved particle' (1725, pp. 226–227). This change was purely conceptual: the experiments reported in 1720 were not changed significantly and the mathematical model stayed the same as well: it still gave Torricelli's law.

The conceptual change, however, had other consequences. It was part of a larger revision of the second edition. This revision was related to the so-called *vis-viva* controversy of the early-eighteenth century, which pitted followers of Newton against followers of Leibniz. As is well known, those who thought they defended Newton argued that forces should be measured in terms of the quantity of motion, proportional to the velocity of a body. Leibniz, however, had argued that forces of bodies in free fall were proportional to the height of their drop, which meant, according to Galileo's well-known results, that force was proportional to the square of the velocity. Leibniz called this quantity *vis viva*, living force (Smith, 2006). Recent historical work has shown that the choice of measure for force had important consequences for what kind of mechanical problems one could solve (Van Besouw, 2017b; Morris, 2018).

's Gravesande was a central player in this controversy, as he switched his interpretation of "force" between the first and second edition of his *Physices elementa mathematica*. In the first edition, he had had largely relied on the notion of the quantity of motion in his discussion of bodies in motion, particularly colliding bodies. "Force" played an important role only in his discussion of statics. In 1722, however, he conducted a series of experiments dropping balls in a tray of clay, measuring the displacement of the clay. These experiments convinced him that the "inherent force" of a body in motion was central to accounting for collision in general. This inherent force can be understood as the potential effect a body could have on other bodies during a collision. "Pressure" on the other hand, was redefined by 's Gravesande as an external cause acting on a body over a certain amount of time. Whereas pressures could generate forces, these same pressures were destroyed by "resistances", which 's Gravesande defined as the opposite of pressures. In doing so, he referred to Newton's action-reaction law.[5]

What does this have to do with the fountains? Simply put, everything. In discussing the efflux problem, the square of the velocity had already been a central measure for almost a century. Therefore, it made sense to identify the problem with the new notion of forces. That is precisely what 's Gravesande did from that point onwards. Importantly, he used the same conceptual framework to discuss the mechanics of fluid as he did to discuss solid bodies.[6] He took this new conceptual framework as a starting point to investigate all kinds of forces and resistances in play in fluids in motion in 1725. One of these was the resistance a body encounters while moving through a fluid, a topic treated before by Newton but on which 's Gravesande devised new experiments.

For these experiments, he used an instrument which has not survived but of which we do have a depiction [Fig. 3]. It consists of an upper trough and a lower trough,

5 For 's Gravesande's most elaborate account, see 's Gravesande, 1742, I, p. 28 ff; for analysis, see Ducheyne & Van Besouw, 2022.

6 Giovanni Poleni (1683–1761) had done something similar in Poleni 1718, see p. 42 on Torricelli's law and pp. 46–57 on living forces; 's Gravesande was made aware of his work briefly after his own experiments of 1722 (Ducheyne & Van Besouw, 2022).

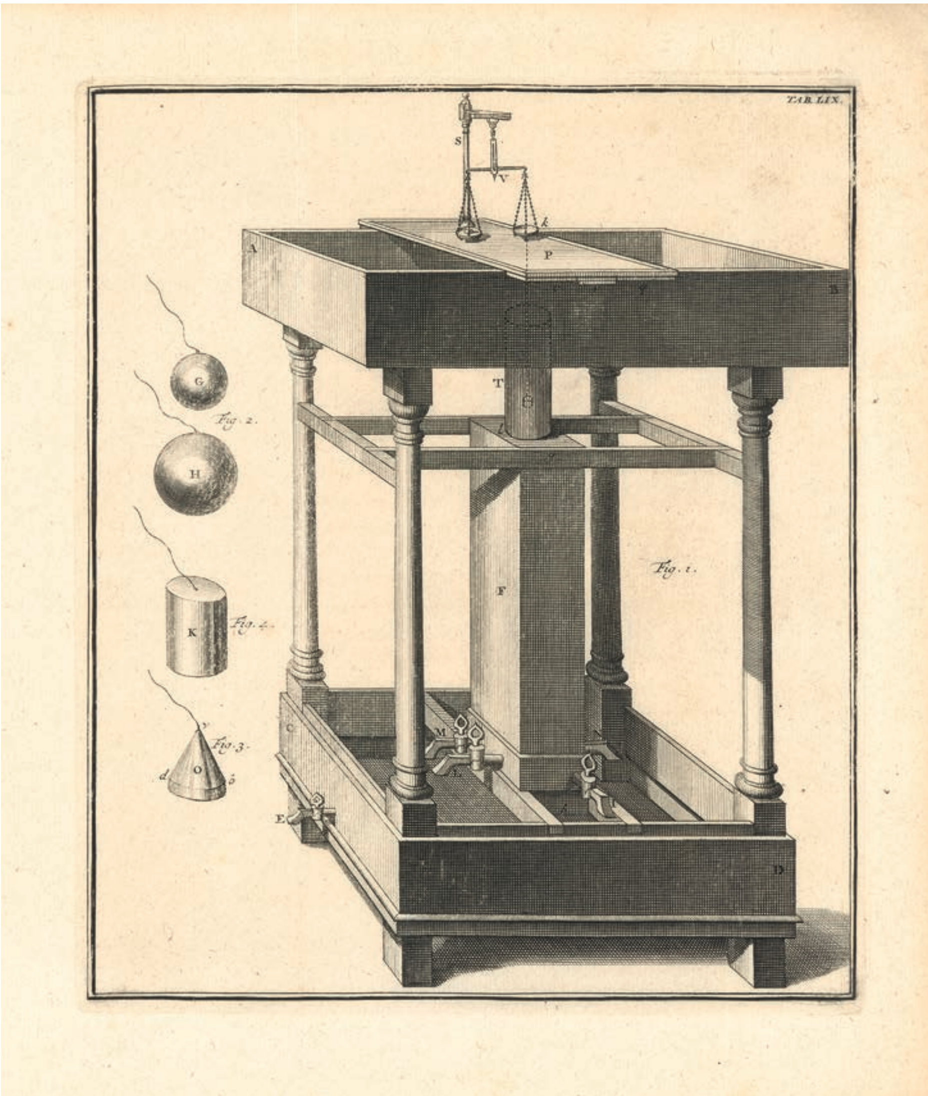

Fig. 3 ’s Gravesande’s instrument to investigate resistance on objects moving through water, ’s Gravesande 1725, plate 35.

connected through a pipe. Water fills the pipe and the lower trough, from which it flows out through a series of cocks. As described by ’s Gravesande, the water of the upper trough is constantly refilled, to have a stable level. The series of valves below functions to closely control the amount of water flowing out: this amount can be set to 1, 2, 3, 4, 5, 6, or 7 times the amount of water that flows out when just the smallest valve is opened (1725, pp. 257–262).

The object on which the resistance is to be measured, is lowered into the pipe hanging from a string. 's Gravesande connected this string to one side of a balance, dragging it down: the amount of weight needed to bring the balance back into equilibrium, was the measure of the resistance encountered. The real trick of this experiment is that it turns the velocity of the water flowing past the object into a parameter that can be varied. This velocity cannot be measured directly, of course. Instead, it is found via Torricelli's principle: by keeping the water level constant and the amount of water flowing out at the bottom in control, 's Gravesande in fact recreated the fountains we have seen before: in the pipe we have a stable column of water, of which all the parts can be taken to move downwards at the same velocity. This velocity, in turn, is directly proportional to the amount of water flowing out. This enabled 's Gravesande to measure the resistance on the object, in terms of weight needed to keep the balance in equilibrium, while varying the velocity from 'one' to 'seven'.

's Gravesande performed this experiment with differently shaped objects, such as cones and balls. The measured resistances were then combined with the mathematical model in which the resistance was proportional to both the velocity squared, due to inertia; and to the velocity simple, due to cohesion. For each object, he looked for the relative influence of each of these two causes to the total resistance (1725, pp. 263–278; 1742, I, pp. 534–547). The exact reasoning he employed is unclear to us, but must have been broadly similar to solving an equation of the form $R = AV + BV^2$, where R is the resistance, V is the velocity of the water flowing past the object, and A and B are constants. According to 's Gravesande, A was a value particular to the fluid in use and B particular to the shape and size of the object. By plugging in the measured values of R and solving the equations for all velocities from 1 to 7 at the same time, the values of A and B can be found. Of interest here is that 's Gravesande was unable to produce values for A and B that would fit all of the experiments: his values were particularly off for the lowest velocity. 's Gravesande attributed this error to the fact that in this case only a tiny weight was sufficient to keep the balance in equilibrium. This weight was about fifty milligrams in modern units, making variation difficult to measure (1725, pp. 263–264). Given how far off the values are, however, 's Gravesande's attribution of the error to a material problem, rather than a conoceptual problem with the model, is remarkable.

In this series of experiments the notion of action, force, pressure, and resistance played a central role, and 's Gravesande concluded that his experiments showed that resistances were proportional to the square of the velocity. Since forces were the opposites of resistances, he argues, forces were proportional to the square of the velocity, too (1725, p. 278). He thus saw his experiments as an intervention in the controversy over forces.

In the final version of this book, of 1742, the chapters just discussed are published without significant change. Further investigations on the "impetus" of water in motion, however, are added. This impetus, 's Gravesande claimed, was the same as the pressure the water could exert on a body, that is, the "resistance" it could overcome while the water lost all its motion. New experiments on the matter again made use of water flowing out of a reservoir with a fixed height. 's Gravesande quantified the impetus of the jet of water by measuring how far this jet could move a pendulum directly in front of the orifice. This pendulum was formed by simply hanging weight from a thread (1742, I, pp. 499–501).[7] In experiments such as these, Torricelli's law is key: it provides quantification for the velocity of the water. The conceptually and mathematically well-defined theorem was thus embedded within the fountains. It was the bedrock of 's Gravesande's hydrodynamical research project. Without it, he could not have quantified resistances and forces.

Modelling river flow

Just as Castelli had intended to do roughly a century earlier, 's Gravesande also used the conceptualization of flow gained from his fountains to model river flow. He considered a river as consisting of flow of water out of a certain basin and assumed that water flowed out as if it were in free fall. This allowed him to calculate the velocity at any point in the river as a function of the height difference between that point and the reservoir ('s Gravesande, 1720, I, pp. 143–147; 1742, I, pp. 479–492). This model of river flow, which was largely the same as Domenico Guglielmini's (Bertoloni Meli, 2006, pp. 181–185; Maffioli, 1994, pp. 262–265), was evidently an enormous simplification of actual river flow. 's Gravesande was aware it had various important shortcomings: he pointed to his neglect of resistance of flow of the riverbed, the inertia of water, and the fact that, in river deltas, the flow of a river is strongly influenced by the tides. He remarked that 'the irregularities in the motion of rivers can vary infinitely, and rules about them cannot be delivered' (1720, I, p. 143; 1742, I, p. 480).

The relation between the model and actual river flows was therefore rather thin. Nevertheless, this conceptual understanding of river flows was in fact transposed back to engineering situations. Together with the surveyor and meteorologist Nicolaas Cruquius (1678–1754), 's Gravesande was asked in the late 1720s and 1730s to advise on the diversion of a river which was threatening to overflow parts of Holland. Their advice was partly based on measurements of actual flow. But these measurements were supplemented by estimates based on 's Gravesande's model for flow velocities. The model, itself based on the experiments carried out with the fountains, thus became a tool to estimate water flow in a real river (Van Besouw, 2024).

7 See also 's Gravesande, 1742 I, p. xxx, where he points out that his experiments do not agree with what others had written on the same topic.

Van Musschenbroek's teaching on projectile motion

Petrus van Musschenbroek was born on 14 March 1692 in Leiden. He came from a well-known family of instrument makers (de Clercq, 1997). We already mentioned his brother Jan, who collaborated with 's Gravesande. Petrus studied at Leiden university and obtained his doctorate in medicine in 1715, under the supervision of Herman Boerhaave.

During his academic career, he held professorships at the university of Duisburg (1719–1722), Utrecht (1723–1739), and Leiden (1740–1761). At each university, van Musschenbroek put his energy into developing or ameliorating the infrastructure necessary for teaching experimental physics. Already in his first year as a professor at Duisburg, an astronomical observatory was built. When he went to Utrecht, he requested that the *Theatrum physicum* that was already in place would be renovated. A few years later, in 1726, a new *Theatrum academicum* was built, again due to requests made by van Musschenbroek. This *Theatrum* would also house the chemical laboratory and anatomical theatre. During his time at Utrecht, van Musschenbroek kept expanding the instrumentarium. In 1740, van Musschenbroek went to Leiden, where he joined 's Gravesande (de Pater, 1979, pp. 24–28). Van Musschenbroek continued teaching at Leiden university until his death in 1761.

His activities as a professor also materialized in the form of several textbooks. The first one was published during his professorship in Utrecht (van Musschenbroek 1726). Throughout his career, van Musschenbroek kept expanding and revising this basic text. This led to the publication of different editions of the textbook, under different titles (van Musschenbroek, 1734; 1741; 1748; 1762a; 1762b). All these editions contain a chapter on projectile motion.

Another important source on van Musschenbroek's teaching are his manuscripts. The manuscript collection held at Leiden University Library contains many lectures in van Musschenbroek's hand.[8] Two manuscripts are directly relevant to the topic of projectile motion. One is dated and was delivered in 1753 (van Musschenbroek, 1753). The other lecture was probably delivered around 1724–1725 (van Musschenbroek, ca. 1724–25).[9] These manuscripts allow us to follow in detail van Musschenbroek's procedure to introduce his students to specific conceptual insights, something that we have been unable to do for 's Gravesande. We will show how van Musschenbroek followed a strict pedagogic method, which we will analyse on the way.

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8 An overview of all the extant manuscripts of van Musschenbroek, including those kept at Leiden University Library, can be found in (de Pater, 1979, pp. 349–371).

9 We base this dating on the fact that the volume of manuscripts in which it is bundled contains a lecture series dated on 1724 as well as on several scattered references to experiments made in the year 1724 and the months December 1724 and April 1725 (van Musschenbroek MS 1724–25, fols 312v, 338v, 340v).

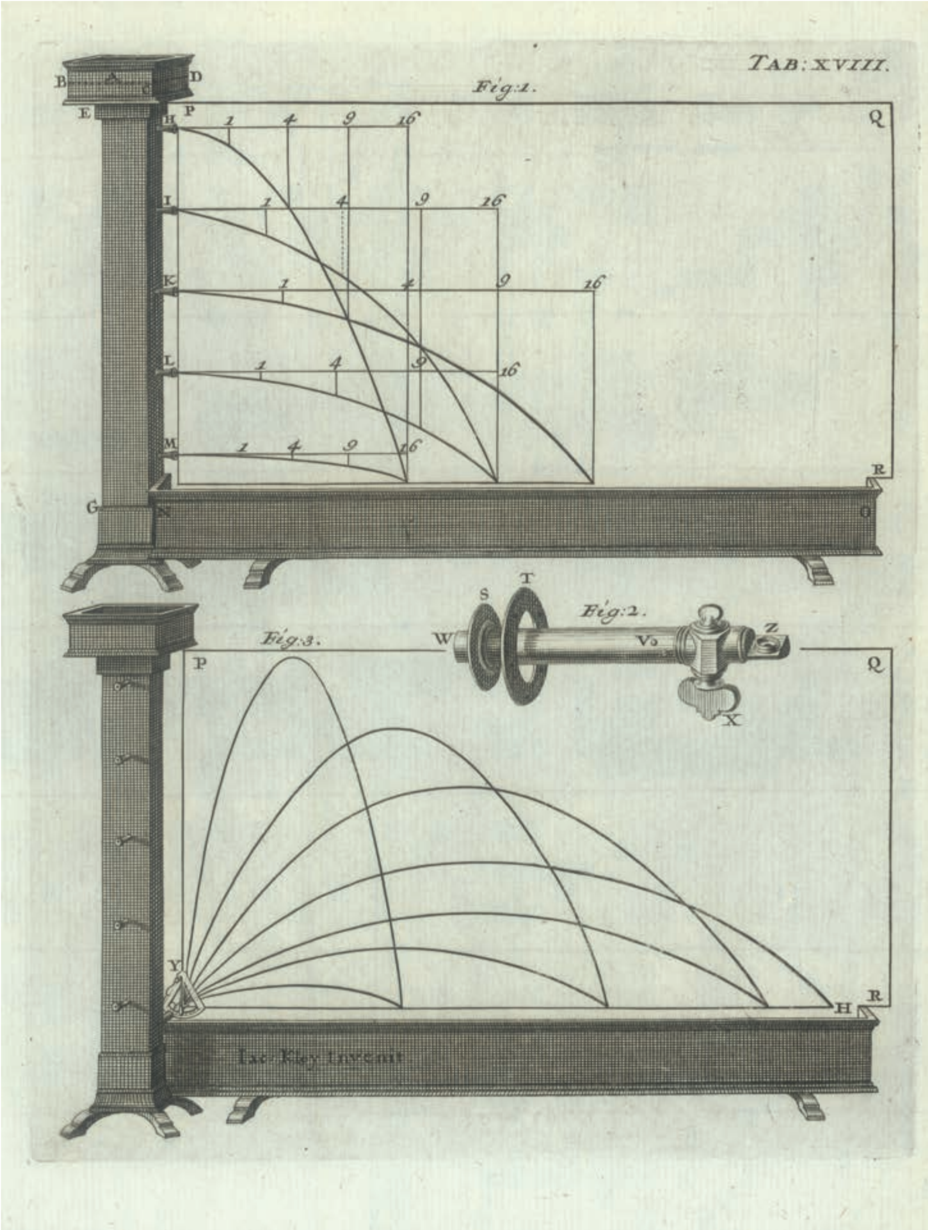

Fig. 4 Depiction of the instrument used in van Musschenbroek's 1753 lecture on projectile motion, as found in van Musschenbroek 1762a. Scan of copy of 1762a in the collection of Ghent University Library. Digital reproduction provided by Ghent University Library.

The main focus of this section will be the teachings of 1753 and, specifically, the fountain in use. This instrument was designed by van Musschenbroek and built by the Rotterdam instrument maker Jacobus Kley. In the final version of his textbook, van Musschenbroek provides a description and image of the instrument (1762a, p. 212). The instrument can be seen as a combination of the "pissing machine" and the mercury fountain discussed above.[10] The central part of the instrument is a wooden column with a lid, both hollow (depicted as EG and BAD in Fig. 4 above). This part of the instrument is still extant and kept at Rijksmuseum Boerhaave (Object V23740).[11] The inside of the lid is wider than the inside of the column.[12] The column is to be filled with water, until the water level reaches the level marked inside of the lid (depicted as a line in BAD in image X below). On the upper part of Fig. 4 we see the first use of the instrument, analogous to the "pissing machines" discussed above. For this, van Musschenbroek uses the five equidistant nozzles H, I, K, L, M), which divide the water column (AG) in five equal parts. The water flowing out of these nozzles follow parabolic trajectories which are depicted on a blackboard (PQR). The blackboards are also kept at the Rijksmuseum Boerhaave (inv.nr. 23746). The through (NO) in which the water was collected has been lost.

On the reverse side of the fountain, there is a spout (WZ in Fig. 4). This was used to show parabolic water jets produced by projecting the water at a certain angle. A quadrant (Y in Fig. 4) was added so that the angle of projection could be visualised. A separate blackboard was used, which depicted the parabolic trajectory followed by the jet when projected at specific angles. To understand the specific uses of the instrument and experiments more generally, however, it is important to take into account van Musschenbroek's remarks and views on the role mathematics plays in natural philosophy, both pedagogically and methodologically.

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10 In the textbook, van Musschenbroek refers to 's Gravesande's mercury fountain and presents his own instrument as an alternative one, based on jets of water (van Musschenbroek, 1762a, p. 212).

11 For the entry on the instrument in the online catalogue of the museum, see https://mmb-web.adlibhosting.com/Details/collect/6368, last access September 15, 2024. According to the catalogue, the instrument is 129 cm high, 45,5 cm wide and 33,5 cm deep. In the textbook, van Musschenbroek only mentions that the column is 4 feet high. Assuming that van Musschenbroek used Rhineland measures, based on the Rhineland rod in Leiden (Gyllenbok, 2010, p. 1800), this corresponds to a height of 125,5 cm. The catalogue remarks that the instrument contains six nozzles, whereas the image in the textbook depicts only five nozzles. To make matters even more complicated, in the manuscript of the 1753 lecture van Musschenbroek says that the instrument is 'perforated in four places' (van Musschenbroek, 1753, fol. 181r). The catalogue also refers to the receipt of the instrument maker, which mentions a price of 60 fl.

12 The catalogue of the Rijksmuseum Boerhaave gives 10 × 11 cm as the inside dimension of the column and 26 × 26 cm as the inside dimension of the lid (see link in footnote 12). It is unclear how this relates to van Musschenbroek's description of the lid as being "four times as wide (*quadruplo amplior*)" as the column (1762a, p. 212).

Mathematics in pedagogy and methodology

Each edition of van Musschenbroek's book contains a different preface. In several of these prefaces, van Musschenbroek commented on the nature of the textbook and the changes that were made in the new edition. These remarks often referred to the pedagogical reasons behind the organization of and changes made to the textbook.

At the beginning of the first edition, van Musschenbroek presents the method followed in the textbook as "the mathematical method" (1726, 'Praefatio', 3b). He further specifies that the knowledge of mathematics is necessary to understand the physics of his days, and to teach, understand, and know its central tenets (idem). In the first chapter of the textbook, it becomes clear that the "mathematical method" is not only understood as a method for scientific practice but should be both understood as a method of scientific practice *and* as a method of presenting a certain subject matter in teaching. The essence of this "mathematical method" is that 'nothing is said in later propositions, which has not been established in previous ones' (2).[13]

As to the content of the work, van Musschenbroek states that he is aware that the work only contains 'the most light (*levissima*)' material, particularly on mathematics, and that he has consciously abstained from adding more difficult material. This he has done because he has

> from experience learned that the tender minds of the students are only capable of learning the most easy principle of physics, and not the more subtle things, which can only be proven with the subtleties of analysis or more profound geometry, and which, when propounded rather deter from physics, than bring their minds to hear it[.] ('Praefatio', 3a-b)

The book itself however does not contain any demonstration, nor does it contain the diagrams which, as we will see, would become so important in the next editions of the textbook. As to the latter, he says that 'without the demonstrations added to them these [diagrams] would be useless,' but and that 'they are drawn during the teaching and demonstrations' ('Praefatio', 6b).

Despite his attempts at keeping the contents as "light" as possible, the contents apparently were not simplified enough. Van Musschenbroek states in the preface of the second edition that he wanted 'to change the earlier order and method somewhat, and to omit those propositions, which could only be proven with longer demonstrations,

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13 As Powers remarks in his study on Boerhaave's pedagogical innovations, in the eighteenth century, "method" not only referred to what we would call "scientific method". There was also a second meaning, 'also known as *ordo*, [which] concerned the proper ordering of topics in a discipline for pedagogical efficiency. Pedagogical method reduced a given discipline (or art) to its precepts or principles and presented them so that each topic provided the student with a foundation of knowledge and skills needed to comprehend the later topics' (2012, p. 117). As an illustration of the importance of *ordo* to van Musschenbroek, we can point to the fact that for several years he lectured on the specific order in which the several disciplines of philosophy should be studied and which books should be consulted for this (Present, 2019).

or which demanded a bit more skill in mathematics' (van Musschenbroek, 1734, 'Praefatio', 4a) He explicitly says that this change was motivated by his teaching experience. A great deal of his students did not have sufficient mathematical background to follow the classes: some of them hardly had a mathematical background at all. Therefore, he was forced to 'use mathematics more sparingly, hardly daring to go beyond the elements, which Euclid had transmitted' (4a). From the second edition onwards, the textbook also contains the diagrams and demonstrations that were previously only communicated to the students during the lectures. Van Musschenbroek says that he did not include these in the textbook at first in order to make sure that the students were more attentive during classes. However, he learned that a lot of students took bad notes because they were unable to follow the mathematical demonstration (5a).

The prefaces of other editions no longer contain remarks on the lack of mathematical background among the student population. This could mean that van Musschenbroek had finally struck the right balance with regards to the mathematical contents of the textbooks. However, the first and second edition were published while van Musschenbroek was teaching at Utrecht university. All subsequent editions were published while he was teaching at Leiden. It could also be that the student population in Leiden was more prepared mathematically than that at Utrecht. If we compare the content of the manuscript of the 1724–25 lecture on projectile motion (van Musschenbroek, ca. 1724–25) delivered at Utrecht university with that of the manuscript of the 1753 lecture on the same topic delivered at Leiden university (van Musschenbroek, 1753), we notice that the latter contains more and relatively more complex demonstrations.

Both in the prefaces to the textbooks and in the lectures, van Musschenbroek comments on the role of experiments as a solution for the lack of mathematical background knowledge amongst the students. In the preface to the first edition, van Musschenbroek writes:

> I will however mention nothing, which I do not also present to the eye and establish by way of an experiment; because in this way a proposition sticks better in one's memory, and whatever weariness the minds of those les fitted for mathematics would have acquired from the demonstration, is taken away by the pleasantness of the experiment. (1726, 'Praefatio', 6a)

In the second edition of the textbook, van Musschenbroek mentions that he had added even more experiments, again 'because these attract and refresh the spectators with their pleasantness, and they can be more easily committed to memory, and remain there the longest' (1734, 'Praefatio', 4b–5a). Following this, he adds that other changes were prompted by recent experimental work and discoveries, both by himself and others (5a). The need to keep the textbook "up to date" is expressed in other editions of the textbook as well. In the fourth edition of the textbook however, it becomes clear that the addition of new insights is still subservient to the overall pedagogical nature of the textbook. This is also the case when it comes to experiments. In the preface of this edition, van Musschenbroek mentions that especially the chapter on electricity has been emendated due to the many discoveries that had been made in the field. He refers to the experiments he performed together with Andreas Cuneus, but says that he only added a selection of them to the handbook:

> It seemed proper to select some experiments from the ones conducted by us, [namely] those which according to my judgment were suitable for this book, and could be rendered in some form of method (*in qualemcunque Methodi formam redigi*). (1748, 'Praefatio', 2a–b)

In order to understand what van Musschenbroek means by this it will be necessary to discuss his views on the role and limits of mathematics in physics. This will also help us to analyse how experiments performed in a research context relate to those performed in an educational setting.

We have seen that in the preface to the first edition of the textbook, van Musschenbroek writes that he will 'which [he does] not also present to the eye and establish by way of an experiment' (1726, 6a). In the 1724–25 lecture on projectile motion, this is indeed the case. Having argued that and demonstrated why projectiles move in a parabolic trajectory, van Musschenbroek says:

> It will be proper to now prove (*probare*) this with a perceptible experiment (*experiment sensibili*), that you may see how nature operates mathematically, and that you may learn that physics is nothing but mathematics, and that the one science cannot be understood or explained if the other is not known. (ca. 1724–25, fol. 392r).

The statement that 'physics is nothing but mathematics' is remarkable given the views that van Musschenbroek had expressed in the inaugural oration that he delivered a year earlier when he took up his professorship at Utrecht. The oration is explicitly presented as an outline of his methodological views (1723).[14] Van Musschenbroek argues that we cannot know the workings of nature through *a priori* reasoning. Only by means of experimentation, and by reasoning on the basis of these experiments, can we gain knowledge of nature (10). Van Musschenbroek does not want to deny the role of mathematics in physics, but emphasizes that we should remain conscious of its limits.

When a mathematician is doing mathematics, he is working within his 'pure science (*scientia pura*)', and his cognition consists in working with ideas about things 'the nature of which he has formed himself [...] and which, being most simple, he conceives in one intuition' (pp. 23–24). This explains the certainty of mathematics. However, whereas in mathematics we are able to conceive a mathematical object in one intuition, this is not possible when we reason about objects in the outside world. When reasoning about bodies, we reason on the basis of ideas representing only certain at-

14 For an analysis of van Musschenbroek's orations from a rhetorical point of view, see Present 2020.

tributes of those bodies. We should therefore be conscious of the difference between the two cases: '[in the one case only] one attribute of a composite thing is contemplated in an abstract meditation, [in the other] the nature of a most simple thing will be contemplated' (pp. 24–25). Reasoning mathematically about physical phenomena therefore does not guarantee that the certainty found in mathematical reasoning is transferred to physics.

We should therefore get rid of the illusion that we are still reasoning mathematically when we are blindly applying mathematics to physics in cases where such an application is not possible. In those cases, van Musschenbroek says, 'we are abusing mathematics' (p. 27). The use of mathematics in physics is limited, because the ideas we have of natural bodies are also limited. Nature is more complex than the abstract ideas we use in mathematical reasoning suggest.

A concrete example of the way this relates to van Musschenbroek's experimental research can be found in his work on the strength of materials. Although Mariotte had performed some experiments on the topic, van Musschenbroek was the first to pursue a systematic and extended experimental investigation into the strength of materials. Before him, Galileo, Mariotte and Leibniz had dealt with questions related to this topic, but they had done so mainly through idealisations, in order to make the behaviour of bodies under stress susceptible to mathematical analysis. In practice, this consisted in the construction of a diagrammatic representation of for example a beam breaking under a certain load.

Van Musschenbroek showed experimentally that the theoretical conclusions that Galileo, Mariotte and Leibniz had reached did not describe what happens in reality. The latter had underestimated the complexity and heterogeneity of materials and had drawn unwarranted conclusions based on an abstract representation of the phenomena. To avoid this, van Musschenbroek constructed an "experimental history", a collection of as much data as possible resulting from a prolonged series of experiments in which relevant variables are systematically varied. The representational tool used by van Musschenbroek in his research is not a diagram, but a table containing numerical results. His research experiments are linked to this representational tool. In his research on the strength of materials, the experimental set-up is designed in such a way that it allows for an easy variation of parameters and the performance of series of experiments in which each experiment produces a numerical output: in this case, a specific weight that breaks a specific material when applied in a specific way. Only after long tables containing these specific data are constructed can one begin to look for regularities (Beck, 2023).

This method clearly prioritises experiments over mathematical modelling. Nevertheless, van Musschenbroek made clear that he did not want to do away with mathematics in general. He did stress that mathematically drawn conclusions need to be confirmed by experiments:

> By means of a perceptible experiment (*experimento sensibili*) we can certainly try to confirm (*comprobare*) what has been concluded by means of mathematical reasoning. If this can be done, then a thing will be shown which such certitude that no doubt will remain. (1723, p. 30)

This describes the order followed in the 1724–25 lecture. After having argued that a projectile follows a parabolic trajectory, van Musschenbroek performs an experiment which shows that this is indeed the case. In both the oration and the lecture, he uses the phrase *experimento sensibili (com)probare*: to prove it with a perceptible experiment. In the 1724–25 lecture, he uses a mercury fountain, 'so that the flow of sprinkling mercury is projected horizontally, and flows down in the parabola depicted on the black table' (ca. 1724–25, fol. 392v). The mercury fountain is probably similar to the one used by 's Gravesande. As we will see below, the blackboard was not just there to confirm what had been demonstrated mathematically. It also helped students link the diagrams they encountered in the textbooks with the phenomena as exhibited in the experiments. As such, the blackboard helped the students grasp the content of the textbooks and played a role in their development of diagrammatic literacy.

Diagrammatic and experimental demonstrations

In this section, we look at the diagrams in van Musschenbroek's chapter on projectile motion in the 1748 edition of the textbook, which was in use in 1753. Here he followed his "mathematical method" meaning that 'nothing is said in later propositions, which has not been established in previous ones'. The diagrams clearly show how van Musschenbroek builds on the knowledge that students have gained by working through the previous chapters.

The chapter first asserts that a projectile is 'being moved by a twofold motion, one due to the projecting cause, the other due to gravity' (1748, pp. 169–170). Given the 'laws of compound motion (*leges de motu composito*)' discussed three chapters before, the projectile 'will always be found in the diagonal of a parallelogram, which is constructed on both directions of the forces (*potentiarum*)' (170). Taking these things as known, van Musschenbroek gives a description of the trajectory of a projectile. The reader is referred to a diagram ('Fig. 4' in Fig. 5).[15] The description takes the form of van Musschenbroek guiding the reader/student through the diagram.

15 The image is a scan of the diagrams found in 1762a. When we refer to diagrams used in the 1748 textbook, the reader can be assured that they are identical to the ones depicted in the 1762a textbook. We use a scan of the 1762a textbook because the diagrams on projectile motion are grouped more together and some extra diagrams are added which are also used in the 1753. As such, this allows us to limit the use of images in this article.

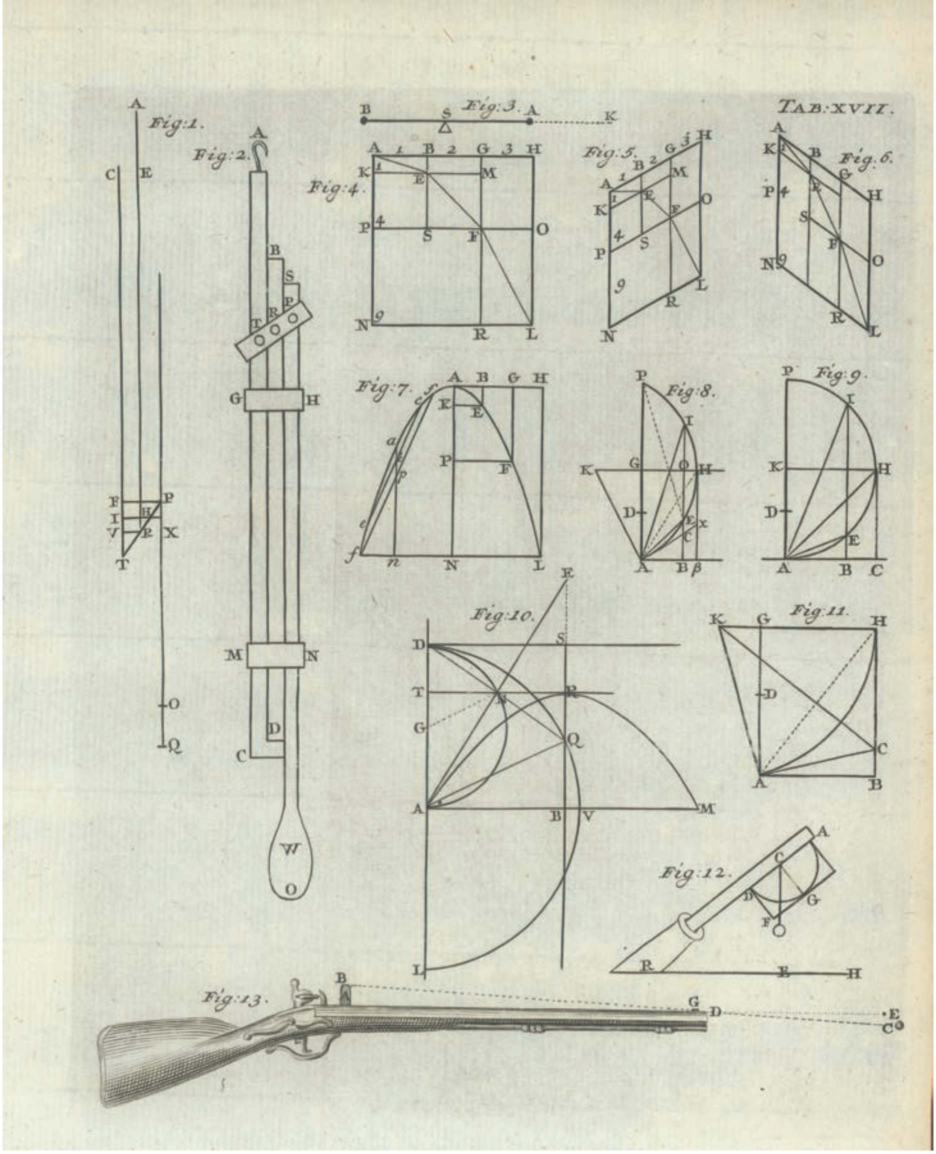

Fig. 5 Diagrams on projectile motion as found in van Musschenbroek 1762a. Some diagrams are added and not found in the 1748 edition of the textbook, but the other diagrams are identical to those in the previous edition. Scan of copy of 1762a in the collection of Ghent University Library. Digital reproduction provided by Ghent University Library.

The reader is asked to imagine a body A (i. e., a body situated in point A) being thrown in the direction AH. The reader therefore knows that the body should be imagined as moving from left to right. The line AH is then divided in equal parts. At this point in the textbook, the reader has already learned several things. She has learned about uniform motion and Newton's first law (1748, pp. 69–70). She will thus also know that a body A 'in equal times will travel equal distances' (p. 69).[16] In that chapter as well, the explanation is coupled to a diagram. The student has learned that bodies are represented as points, and that motion is therefore represented as a line (p. 67). This is coupled to a simple diagram consisting of an arrow pointing right, divided in equal segments (AC, CD, DE, EF, FB). To a modern educated reader, this diagram might seem somewhat redundant, but it shows how van Musschenbroek systematically tries to develop the diagrammatic literacy of his students. He does this by showing how the simple diagram depicting uniform motion can also be read in another way. In following the movement of a body from A to B in the arrow, van Musschenbroek says, this movement should also be read as implying a passage of time (p. 67). Given that a body moving with uniform motion (such as body A in the diagram on projectile motion) describes equal spaces in equal times, the line A H divided in equal parts in Fig. 4 can also be read as representing the passage of time (p. 61).

After this, van Musschenbroek introduces the downward motion of the projectile due to gravity. In chapter 7, the student will have learned about gravity. She now knows that a falling body is moved by an accelerated motion. She will also have learned that 'the distances travelled by [the heavy body] in equal times are as the odd numbers 1, 3, 5' (p. 101). This had been proven by means of the so-called triangle of speeds.[17] In chapter 10, the student has learned about the aforementioned "laws of compound motion", which again included learning how compound motion can be represented diagrammatically (pp. 146–153). In each case, the teaching of the relevant concepts and theories was combined with a discussion of diagrams. In having worked through the

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16 Due to our use of the female pronoun, one of the reviewers asked whether there is any information about women attending van Musschenbroek's lectures. We did not find any references in the literature to female students attending his lectures. The first female university student in the Dutch Republic (and Europe) to attend lectures was Anna Maria van Schurman (1607–1678) (Larsen, 2016). But she remained an exception for a long time. As Maria Rosa di Simone notes, it was only in the nineteenth century that real progress was made with regard to female emancipation in the universities (1996, p. 295) She also mentions how the few exceptions in the seventeenth and eighteenth century, such as van Schurman, caused a big stir and debate in European culture (1996, p. 296). We can therefore assume that van Musschenbroek's audience was strictly male. This does however not mean that this was also the case for his readership. In the eighteenth century, in the attempt to popularize "Newtonian" natural philosophy, "Newtonian" books were written for and by women. Unfortunately, these books still utilised gender stereotypes by presenting "soft" versions of the natural scientific content, and often contained explicit reassurances that the learning contained in the books will not make women less fit or willing to perform their domestic duties (Hutton 2017). As to van Musschenbroek's textbook, Émilie du Châtelet (1706–1749) can be mentioned as a famous female reader of van Musschenbroek's textbooks. Du Châtelet corresponded with van Musschenbroek and also criticised certain passages in his textbooks as a means to present her own natural philosophical views (Brading & Lin, 2023).

17 On the development of the triangle of speeds and Galileo's use of diagrams in his mathematization of motion, see Palmerino 2010. Especially relevant in this context is her claim about the complementarity of mathematical demonstration and construction of the diagram (pp. 446–447).

preceding chapters, the student will have acquired the necessary diagrammatic literacy to follow the construction of the diagram representing the motion of a projectile.

However, the diagram is not yet a precise representation of projectile motion. Van Musschenbroek next asks his reader to mentally make the time intervals on AH smaller and smaller. This would make the parallelograms in the diagram smaller and smaller as well and would make the line into a curve. Van Musschenbroek then argues that this curve is a parabola. First, he refers to a diagram representing a parabola ('Fig. 7' in Fig. 5). Without providing a further demonstration, he takes it to be mathematically demonstrated that one of the properties of a parabola is that if AN is the axis of the parabola, and KE and PF are ordinates of the same parabola, then AK is to AP as KE^2 is to PF^2. He says that this is also the case in the trajectory of the parabola, because in the diagram we can see that AK is to AP as KE^2 is to PF^2, which is the same as saying that AK is to AP as AB^2 is to AG^2.[18]

'Fig. 4' is constructed on the basis of physical knowledge: knowledge of the nature of uniform motion, the motion of falling bodies, and the composition of motion. In the chapters where this knowledge was presented to the student, the presentation was backed with empirical observations. That is, given the empirical observations underlying the knowledge that went in the construction of the diagram, we can assume that the diagram represents the motion of an actual projectile. The diagram can thus be seen as a "physical diagram".

'Fig. 7', on the other hand, is at first presented as a purely mathematical diagram. That is, it represents an abstract, mathematical object. As we have seen, van Musschenbroek warns against assuming that abstract mathematical ideas can immediately be applied to physics. In this case, however, he explicitly argues that the mathematics and physics "match". Given the nature of the trajectory of a projectile, based on physical properties, treating the trajectory as a parabola is warranted. This, he asserts, allows us to use mathematics to infer further information from the diagram. To use van Musschenbroek's words, once the match is demonstrated: 'The parabola can be used to determine the motion of projected bodies in a vacuum: which is the foundation of the art of ballistics' (p. 170). Van Musschenbroek then uses the diagrams to demonstrate certain propositions which are useful in this context. He shows that a projectile can be thrown furthest when it is thrown at an angle of 45 degrees, how to determine the speed with which a projectile should be thrown, given that all the other relevant data are given, and how to hit a target with the minimal amount of speed (pp. 172–173).

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18 This is consistent with the fact that the trajectory of the parabola is taken to be a compound of its horizontal uniform linear motion and its downward accelarated motion. Based on the former, we know that AB and AG are equal. Ergo AB^2 is to AG^2 as 1 is to 4. AK represents the distance that the body will have fallen at point B, AP represents the distance that the body will have fallen at point G. The distances travelled are as the squares of the time, so given that B is time 1 and G is time 2, AK will be one and AP will be 4. Ergo AK is to AP as AB^2 is to AG^2.

The 1753 lecture

The relation between mathematics and experiment, through the uses of diagrams, can be further clarified by looking at the 1753 lecture. Van Musschenbroek begins it by referring to the utility of the topic of projectile motion not only 'in the art of hunting, but also in the art which deals with the jets of water, which are expelled from fountains, and in [the art of]? war' (van Musschenbroek 1753, fol. 176r). He then immediately refers to the experiments that he will perform:

> I have decided to lay down the foundations of all these [topics] and to confirm my assertions with experiments, so that, while I am not to be taken to suppose that you, my hearers, are absolutely ignorant of mathematics, the experiments seen will lessen the weariness which would trouble the minds of people that are less accustomed to mathematical demonstrations. I will thus try to be useful and to amuse at the same time. (176r)

Van Musschenbroek then provides a short rehearsal of Galileo's law of the fall (176v–177v) but afterwards follows the order of the textbook. He constructs the diagram representing the motion of a projectile and demonstrates that the trajectory has the properties of a parabola (178r–179v). He then confirms this with an experiment. For this experiment, he does not use the instrument discussed at the beginning of this section, but a demonstration device with a ball rolling down a ramp and then falling through several rings positioned in the form of a parabola.[19] In the remainder of the lecture however, van Musschenbroek uses the machine depicted in Fig. 4.

The transition from the instrument with the rolling balls to the water-spouting instrument is justified, he claims, since 'it is the same whether a solid body is projected or a fluid', because a jet of a fluid is nothing more than a collection of particles succeeding one another. Therefore, a projected jet of fluid will also form a parabola (180v). Van Musschenbroek demonstrates this by using the side of the instrument with the different nozzles. Van Musschenbroek uses the nozzles one by one. In conjunction with the blackboard (in the way depicted in 'Fig. 1' in Fig. 4), van Musschenbroek can provide a visual demonstration of why the trajectories followed by the jets of water are indeed parabolic, analogous to the argumentation discussed above.

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19 Such an instrument is described and depicted in 's Gravesande's textbooks. We have reasons to suspect however, that van Musschenbroek had also designed and commissioned a new version of this instrument from Kley. The collection at the Rijksmuseum Boerhaave holds an instrument constructed by Kley which involves a ball rolling down a slope and moving through hoops positioned in a parabola (Object number V13994, https://mmb-web.adlibhosting.com/Details/collect/6340, last access September 15, 2024). The catalogue mentions 's Gravesande as a designer and states that Kley made the instrument between 1750 and 1787. They add however that the dating and the attribution are "problematic". One of van Musschenbroek's manuscripts, which not only contains several designs and descriptions of instruments, but also contact information of instrument makers and reminders to pay people (including Kley), contains a fragment which seems to describe parts of these instruments (van Musschenbroek, n. d., fol. 201r).

Having given this demonstration, van Musschenbroek then tells his students that 'the fluid is pushed out of the openings with the same speed that a heavy [body] acquires when falling from an altitude which is equal to the straight [line] drawn from the surface of the water up until the location of the hole' (181v). This is exactly the proposition from which Torricelli started in order to formulate his law. A bit later, van Musschenbroek will formulate this law to his students, without however referring to Torricelli (183r). Based on this principle, van Musschenbroek demonstrates that

> a parabola with the biggest amplitude is transversed, when a fluid springs forth from [a nozzle situated at] half the height of [the surface of] the fluid, and that parabolas with the same amplitude [are transversed] when the holes are in positions equally higher or lower from the middle of the [column of fluid]. (185v)

As one can see inFig. 4, 'Fig. 1', the instrument and the blackboard are again designed to enable van Musschenbroek to demonstrate this *ad oculum*. The parabola with the biggest amplitude is produced by the jet of water coming from the middle nozzle K. The parabolas coming from nozzles I and L and H and M respectively have the same amplitude and touch the horizon at the same places.

It should be emphasized that van Musschenbroek nowhere presents the lecture as even partly dealing with hydrostatics or fluid dynamics. As we just mentioned, Torricelli's law is formulated without even mentioning Torricelli. And, in fact, van Musschenbroek reverses the order found in Torricelli's original. Whereas Torricelli used projectile motion to conceptualize efflux, van Musschenbroek used Torricelli's conceptualization of water outflow as a starting point for his treatment of the parabolic shape produced by his instrument. At first sight, the most obvious reason one could think of as to why van Musschenbroek designed this specific instrument, is that it allowed him to use one and the same instrument to show several different effects. We would like to argue, however, that his use of Torricelli's law points towards a specific visual use of the instrument: illustrating the diagrams present in geometrical demonstrations about ballistics.

In fact, the instruments pops up in the discussion of several propositions related to parabolic trajectories (see 'Fig. 3' in Fig. 4). Again, van Musschenbroek starts by using the instrument to substantiate the claim that the trajectory followed by the water is indeed parabolic. For this, he uses the other side of the blackboard. This side is not depicted in the textbook, but can be seen in Fig. 6. A line depicts a direction of thirty degrees. This line is divided in equal parts. From the end of each part a line is drawn perpendicularly to the horizon up until the line representing the trajectory of the jet of water. These lines are to each other as 1, 4, 9, 16, as depicted on the board.

In the lecture, van Musschenbroek uses diagrams ('Fig 10' and 'Fig. 8' in Fig. 5) to demonstrate several propositions. One of them is that the amplitude of the parabola produced by a jet of water is largest when the water is projected at an angle of 45 degrees. A related proposition is that when two angels differ equally from 45 degrees, the

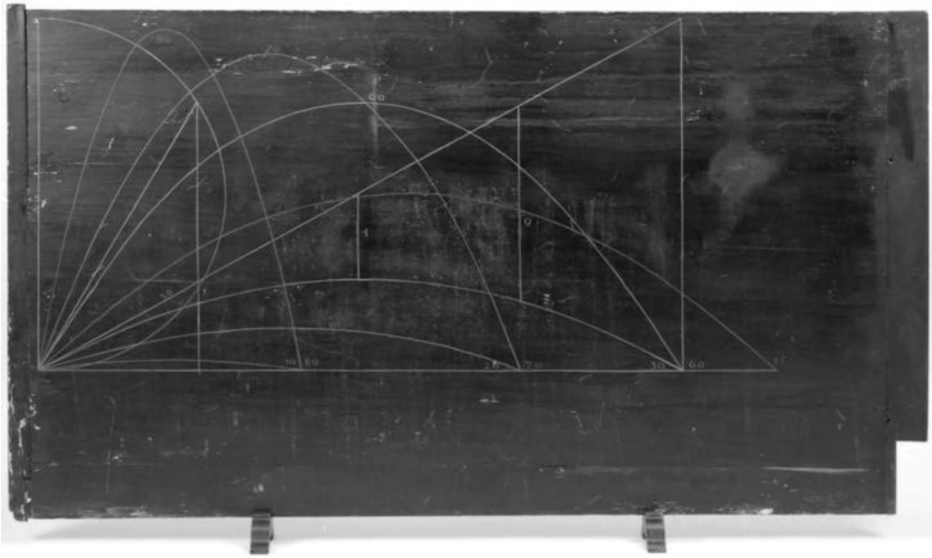

Fig. 6 Blackboard used by van Musschenbroek. Photograph provided by Rijksmuseum Boerhaave

amplitude of the parabola produced by jets of water projected at these angles will also be the same. On the blackboard we see different pairs of parabolas meeting the horizon at the same point. The angles at which the water jets were projected in order to produce these parabolas are depicted at these points. In this way, van Musschenbroek can confirm and demonstrate *ad oculum* that water jets projected at an angle of 10 or 80, 20 or 70, and 30 or 60 respectively, all have the same amplitude. The waterjet projected at an angle of 45 degrees clearly has the largest amplitude.

Other propositions deal with the relation between the speed of the projectile, the angle of projection, and the amplitude of the parabola. In the lecture, these propositions are demonstrated using the diagrams depicted in Fig 5 as 'Fig. 8' and 'Fig. 10'. One line is drawn at an angle of 60 degrees and corresponds to AI in 'Fig. 8' and AE in 'Fig. 10'. This can be used to demonstrate how this geometrical construction can be applied to a concrete situation. Having geometrically demonstrated by means of 'Fig. 10', for example, how the amplitude of the parabola can be calculated on the basis of the angle of projection (assuming a certain velocity), van Musschenbroek shows how this should be calculated in practice and immediately demonstrates that the amplitude of the jet of water indeed corresponds to the calculations, granting a small deviation due to the resistance of the air (1753, fols 192r–195r). Here, not only the blackboard, but also the instrument itself plays a role in linking the experimental demonstration to the geometrical demonstration. Assuming that the velocity of the jet of water is proportional to the square root of the height of the surface of the water, van Musschenbroek can *show* that the velocity of the jet remains the same by making sure that the water level is the same in all relevant experiments. The instrument also makes it possible to make a connection with the diagrams in another way, which would not have been possible (or at least not as visibly and elegantly) by means of 's Gravesande's mercury jet or the instrument with the rolling balls. In the geometrical demonstrations, the velocity of the projectile is depicted by means of a line representing the velocity that the body would have acquired if falling from that height (DA in both 'Fig. 8' and 'Fig. 10'). Given that van Musschenbroek had already used Torricelli's law in his discussion of the working of the instrument, the instrument itself becomes a means of visually presenting the understanding of the velocity of the projectile as embedded in the diagram. This allows him to use the experiment to show to his students 'how nature operates mathematically'.

Conclusion

The fountains we have discussed in this chapter were certainly much more than machines to show Torricelli's law to students with a lack of mathematical skills. From Torricelli's own treatment onwards, their importance lied in making conceptual connections between the behaviour of fluids and that of solid bodies in free fall. Whereas Torricelli aimed for an idealized mathematical understanding of both phenomena, however, Huygens, Mariotte, and Newton were very much concerned with the actual mechanics of water, and with phenomena of resistance in particular. They thus broadened the scope of the phenomena investigated with the fountains.

In the section on 's Gravesande, we have seen that he showed fairly little interest in demonstrating Torricelli's law. He did demonstrate it, but mostly so that he could take it for granted and use his fountains for other purposes afterwards. Presupposing that Torricelli's law indeed gave the efflux velocity was the bedrock of his further investigations into the forces, pressures, and resistances involved in the motions of water. It was through Torricelli's law that his fountains could become research instruments in the first place: without it, there was no way of quantifying the velocity of flow, let alone quantify resistances. It was the combination of his conceptual framework and experiments that allowed him to quantify discrepancies, which he would then attribute to various forms of resistance.

Van Musschenbroek, on the other hand, turned the relation between free fall and fountains on its head. Instead of using free fall to solve the efflux problem, he adopted efflux as the best way to visualize parabolic shapes. This he did for pedagogic purposes, and specifically to connect mathematical procedures with physical phenomena. Diagrams were the key mediator between the two, and comparing the jets produced by his fountain with the diagrams on the blackboard, was his key move in creating the connection. Clearly, the fountains produced conceptual clarification in many different ways. They are truly object of understanding, creating connections and allowing their users to expand their knowledge based on whatever it is about mechanics, fluids, or mathematics they already knew.

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Jip van Besouw is a postdoctoral researcher in the project DEEPMED, at the University of Sevilla, where he works on the sciences of deep-sea environments in the early modern period. He is preparing a monograph on Willem Jacob 's Gravesande and the institutionalization of physics post-Newton; he has also worked on the history of mechanics, relativity theory, and river hydraulics.
Jip van Besouw, jbesouw@us.es, **ORCID:** 0000-0003-3138-5233

Pieter T. L. Beck is an FWO Junior Postdoctoral Fellow at the Department of Philosophy and Moral Sciences (Ghent University). He obtained his doctorate at the Vrije Universiteit Brussel in 2019 with a dissertation on Petrus van Musschenbroek. His current project centres on the performance and interpretation of chemical experiments in the late 18th century Dutch Republic, with a strong focus on their use in teaching. The research on this paper was made possible by a grant from the Research Foundation – Flanders (FWO) (project nr 12B4623N).
Pieter T. L. Beck, Pieter.Beck@UGent.be, **ORCID:** 0000-0003-4788-4715